FURTHER MATHEMATICS

Paper 9231/11
Paper 11

Key messages

- Candidates need to make sure that they communicate their working in detail, making sure they fully justify their answers.
- Candidates should read questions carefully so that they answer all aspects in adequate depth. They need to take great care copying information across to their answer booklets, and due notice where exact answers are required.
- Candidates should ensure that any sketch graphs are fully labelled and carefully drawn to show behaviour at limits.

General comments

The majority of candidates were able to attempt all questions on the paper with some success, and a small proportion offered answers to both alternatives of **question 11**. Time did not seem to be a problem. Algebraic handling was generally very good, and most candidates demonstrated sound calculus techniques. Sometimes candidates did not fully justify their answers and jumped to conclusions without justification, particularly where answers were given within the question. There were many scripts of an extremely high standard.

Question 1

Most candidates tackled this successfully using the suggested substitution, although there were a few errors in manipulation. Others chose to form a new equation using the roots, and were also successful, though this method took much longer.

Answer: $7y^3 - 21y^2 + 20y - 8 = 0$

Question 2

Although the majority of candidates split the expression into its three component fractions correctly, some then went on to recombine the terms. There were some errors in cancelling out the terms, usually caused when candidates did not write down the full series. Most students were able to find the sum to infinity for their expression.

Answers: **(i)** $1 - \frac{2}{n+1} + \frac{2}{n+2}$ **(ii)** $S_\infty = 1$

Question 3

The majority of candidates opted to look at the difference between successive terms, and were able to argue the divisibility case well. Some successfully manipulated f(k+1) to find an expression in terms of f(k), but this route was more difficult. In most cases the language of an induction proof was well expressed. However, some candidates did not use fully correct phrases to explain the steps.

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Question 4

This question caused some problems, and some candidates took a long time to eliminate r and θ from the expression, whilst others never reached a Cartesian equation. Sketches sometimes included a section in the third quadrant i.e. outside the correct domain and range. Many candidates forgot to apply the change of variables when using the given substitution. Some ignored the substitution and struggled with the integration although a few found an alternative method successfully.

Answers: **(i)** $xy = 4$ **(iii)** $2 \ln 3$

Question 5

Where candidates used the given differentiation as instructed, they were usually able to spot the necessary manipulation of $\cos^2 x$ and complete the proof. However, some candidates left out steps of working and, because the answer was given, they lost marks. The majority of students successfully found I_0, and others used trigonometric manipulation to calculate I_2

Answers: **(ii)** $I_4 = \dfrac{\pi}{32}$

Question 6

The majority of candidates used de Moivre's theorem successfully to find $\cot\theta$ with just a few struggling to translate their expression in cos and sin into an expression in cot. The second part of this question proved more challenging, and only the strongest candidates appreciated the necessary lines of thought. Many dropped the solution with $k = 7$ without comment, and the product of roots was not matched to the equation accurately. Few candidates made the essential pairing of the roots explicit.

Answer: $\cot 7\theta = \dfrac{\cot^7\theta - 21\cot^5\theta + 35\cot^3\theta - 7\cot\theta}{7\cot^6\theta - 35\cot^4\theta + 21\cot^2\theta - 1}$

Question 7

Although most candidates embarked on polynomial division to find the oblique asymptote, many failed to complete the division. Greater success was evident in the vertical asymptote. Candidates who used the discriminant of $x^2 - yx + 2y = 0$ to find the range mainly gave good evidence, although some reversed the inequality without due explanation. Others offered incomplete arguments referring to the maximum and minimum points or to the graph. The curve was mainly well sketched, with relevant points clearly marked, although some did not show correct forms at infinity.

Answers: **(i)** $x = 2$ and $y = x + 2$ **(iii)** Maximum at (0,0) and minimum at (4,8).

Question 8

This was a very successful question for most candidates, who were well versed in the necessary vector techniques. Unfortunately some misread or miscopied the coordinates at times, losing themselves accuracy marks. Some also made calculations more difficult by failing to divide down their direction vectors. A few candidates did not appreciate that the coordinates of a point cannot be replaced by a multiple of themselves.

Answers: **(i)** $x + 2y + z = 3$ **(ii)** $\theta = 70.9°$ or 1.24 radians **(iii)** $\mathbf{r} = \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix} + t\begin{pmatrix} 5 \\ 1 \\ -7 \end{pmatrix}$ or equivalent.

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Question 9

Many candidates tackled this question confidently, often beginning with the complementary function. Some did not realise that they only had to calculate *k* for the particular integral, and spent time checking other possible particular integrals. Candidates who found the general solution were usually able to use the boundary conditions to complete the question successfully.

Answers: **(i)** $k = 2$ **(ii)** $y = Ae^{2x} + Bxe^{2x} + 2x^2e^{2x}$ **(iii)** $y = 3e^{2x} - 8xe^{2x} + 2x^2e^{2x}$

Question 10

Many candidates did not recognise that the eigenvalues of this matrix could be read from the diagonal and spent much time calculating them. Once they were found, they were able to find the corresponding eigenvectors. Candidates would have benefited from reducing their vectors to the form below to save calculation of $\mathbf{P}^{-1}$. Candidates were able to identify the correct calculation for $\mathbf{A}^n$ but were not always precise with their use of brackets when multiplying together the matrices.

Answers: **(i)** Eigenvalues –2, –1, 1 Eigenvectors are $\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}$ or equivalent

$$\mathbf{P} = \begin{pmatrix}1 & 1 & 0\\0 & 1 & 1\\0 & 0 & 1\end{pmatrix}, \mathbf{D} = \begin{pmatrix}-2 & 0 & 0\\0 & -1 & 0\\0 & 0 & 1\end{pmatrix}$$

$$\mathbf{A}^n = \begin{pmatrix}(-2)^n & -(-2)^n + (-1)^n & (-2)^n + (-1)^{n+1}\\0 & (-1)^n & (-1)^{n+1} + 1\\0 & 0 & 1\end{pmatrix}$$

Question 11

EITHER: More candidates chose this option, and most completed the first part very well, with just a minority making careless mistakes in differentiating. Candidates were mainly secure in recalling the correct formula for arc length. The simple integration required to complete the first part did not cause problems, but in the second part many candidates had difficulty finding the correct reduction formula. Most set about the integration correctly, but made minor errors with signs or with multipliers leading to errors in the final answer.

Answers: **(i)** $\sqrt{2}\left(e^{\pi} - e^{-\pi}\right)$ or 32.7 **(ii)** $\frac{2\sqrt{2}\pi}{5}\left(e^{2\pi} - e^{-2\pi}\right)$ or 952

OR: Although this was less popular, for those that chose this option it very often proved a successful end to the paper. Candidates performed the necessary row reduction and were able to find the rank of the matrix **M,** and go on to find a basis for the null space of the transformation. The multiplication was well done. The final part caused some confusion with weaker candidates and they found the algebra difficult to complete without slipping up.

Answers: **(i)** r(**M**) = 2 **(ii)** $\left\{\begin{pmatrix}1\\0\\1\\1\end{pmatrix}, \begin{pmatrix}2\\1\\0\\0\end{pmatrix}\right\}$ **(iii)** $\begin{pmatrix}15\\33\\66\\81\end{pmatrix}$ **(iv)** $\mathbf{x}' = \begin{pmatrix}4\\-1\\3\\0\end{pmatrix}$

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FURTHER MATHEMATICS

Paper 9231/12
Paper 12

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$$\mathbf{A}^n = \begin{pmatrix}(-2)^n & -(-2)^n + (-1)^n & (-2)^n + (-1)^{n+1}\\0 & (-1)^n & (-1)^{n+1} + 1\\0 & 0 & 1\end{pmatrix}$$

Question 11

EITHER: More candidates chose this option, and most completed the first part very well, with just a minority making careless mistakes in differentiating. Candidates were mainly secure in recalling the correct formula for arc length. The simple integration required to complete the first part did not cause problems, but in the second part many candidates had difficulty finding the correct reduction formula. Most set about the integration correctly, but made minor errors with signs or with multipliers leading to errors in the final answer.

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FURTHER MATHEMATICS

Paper 9231/13
Paper 13

Key messages

- Candidates need to make sure that they communicate their working in full, particularly in questions where the answer is given on the question paper. They also need to present their solutions in a structured manner, taking care over their presentation.
- Candidates should take advantage of suggested methods to find efficient solutions to problems
- Candidates should read questions carefully and answer all aspects in adequate detail.
- Candidates need to check that they have given full justification of statements made.

General comments

The paper allowed candidates to display their knowledge of the syllabus to good effect. They were able to attempt all questions, with a small proportion offering answers to both alternatives of **question 11**. Algebraic handling was generally sound, but a number of candidates failed to apply common calculus techniques accurately. There were times when candidates tried to apply complex formulae from memory. This was not always a successful strategy. On the whole there were very few misreads, and calculation slips. There were many scripts of an extremely high standard.

Comments on specific questions

Question 1

Most candidates chose to split the algebraic fraction rather than verify the given answer and found the numerators efficiently. Some failed to give enough evidence. Apart from occasional loss of the multiplier, $\frac{1}{3}$, the second part of this question was well done, with candidates finding the sum to infinity correctly. The inequality was well handled.

Answer: $N > 1109\frac{7}{9}$. Least $N = 1110$.

Question 2

This question proved challenging, and a number of candidates offered a deductive proof. Others approached the question purely algebraically, and failed to refer to the context. The language needed to construct an induction proof was not always used, or statements lacked clarity and detail.

Question 3

Candidates who used the determinant to solve this problem generally offered concise and accurate solutions to the first part, finding the two values of k from their cubic equation. In general, those who used row manipulation were less successful at finding two solutions. Although most candidates were able to show that when $k = 1$ the equations were inconsistent, a number allocated a value to one of the variables when $k = 2$, and did not find the general solution.

Answer: $k = 1$: inconsistent, $k = –2$ then $(x,y,z) = ([–1 + t], t, t)$ or equivalent

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Question 4

The first two parts of this question were well done but a number of candidates made sign errors when integrating $\ln(1-x)$. Some candidates did not simplify their answers sufficiently and others offered numerical answers without showing any working despite the need for an exact answer.

Answer: **(iii)** $2\ln 3 - 1$

Question 5

Although most candidates were able to differentiate the given expression, a substantial number stated that the result was negative without sufficient justification. Those that split the expression into partial fractions before differentiating benefited from the extra work. Stronger candidates realised the need to complete the square or support their arguments. The two vertical asymptotes were identified successfully, but many forgot the horizontal asymptote. Sketches were generally well done, with coordinates and asymptotes clearly marked.

Answers: **(i)** $\frac{dy}{dx} = \frac{-(x+2)^2 - 5}{(x^2-9)^2} \Rightarrow \frac{dy}{dx} < 0$ **(ii)** Asymptotes: $x = \pm 3$; $y = 0$

Question 6

Many candidates followed the guidance and were able to split the resulting expression - $(4-x^2)^{\frac{3}{2}} = (4-x^2)(4-x^2)^{\frac{1}{2}}$ which allowed them to integrate successfully. A number did not follow through the integration fully, but a majority tackled this quite complex reduction formula efficiently. Candidates who tried to find a reduction formula without using the suggestion struggled. Unfortunately not all candidates recognised the simple integration required to find I_1 and although some found successful substitutions, many spent some time trying to integrate this expression.

Answer: $I_3 = \frac{64}{15}$

Question 7

This question proved challenging for many candidates who were unable to fully eliminate θ – attempts at half angle formulae were time-consuming and not productive. There seemed to be quite a widespread lack of confidence in moving between polar and Cartesian coordinates. Although more candidates managed to sketch a parabola, many omitted to label the intercepts with the axes. A wide number of different curves through the correct intercepts were also seen. Even where candidates had found the correct Cartesian equation in **(i)**, they found it hard to identify the correct integration for part **(iii)**. Despite the wording of the question, they attempted to integrate the polar expression.

Answers: **(i)** $y^2 = 1 + 2x$ **(ii)** Parabola symmetrical about *x*-axis. Intercepts $\left(-\frac{1}{2}, 0\right)$ and $(0, \pm 1)$

(ii) Area = $\frac{2}{3}$

Question 8

This question was mainly well done, although not all candidates who attempted to use a formula, for example for $\Sigma\alpha^3$, remembered it correctly in part **(i)**. They might have had more success tackling the summation using the equation itself, although some managed to manipulate expressions to a successful conclusion. Part **(ii)** was well done. Candidates were required to justify the given substitution in **(iii)**, which many omitted, and this was one of the few questions were algebraic errors were relatively common. Some candidates attempted to transform the given equation in all three parts, but this was rarely time-effective or indeed successful.

Answers: **(ii)** $S_4 = 27$ **(iii)** $5x^3 - 16x^2 + 18x - 6 = 0$

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Question 9

Candidates who expanded $\left(z \pm \frac{1}{z}\right)^4$ in the first two parts were rewarded with a concise route to the correct answer. Others were able to work through the expansion of $(\cos\theta + \mathrm{i}\sin\theta)^4$ and subsequent use of double angle formulae, but this method was more prone to errors. A number of candidates were penalised because they did not use de Moivre's theorem. Some numerical answers to **(iii)** appeared without working, which did not earn marks.

Answer: **(iii)** $\frac{1}{16} + \frac{3}{32}\pi$

Question 10

Whilst most candidates were able to apply the chain rule to the first part of this question, not all of the many methods used for the second differential expression were valid. There was some inaccurate use of the derivative notation, and some switching between the three variables. Clear handwriting was essential so that x, y and u were not confused. Confusion between variables was also seen in **(ii)**, though the majority worked neatly and correctly through the steps to finding the particular integral and the complementary function. A few more problems were seen in applying the boundary conditions, with some of these problems caused by poor differentiation.

Answer: **(ii)** $y = 2\ln x + 1 - \frac{1}{x}\left(\cos[4\ln x] + \sin[4\ln x]\right)$

Question 11

EITHER: This was the preferred option, and most candidates produced accurate work to find the eigenvectors and final eigenvalue. Some went to the unnecessary lengths of finding the 3 eigenvalues before tackling the question set. There were some errors in finding the inverse of matrix **P**, and candidates might be well advised to simplify their eigenvectors to make this process easier. The final part of this question proved more challenging, and few candidates saw it through to a successful conclusion.

Answer: $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ (or equivalent) $\quad \lambda = 9$

$$\mathbf{P} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ -1 & -2 & -1 \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{pmatrix} \quad \mathbf{P}^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & -1 \\ -1 & 1 & 0 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 1 & -1 & -1 \\ -2 & 2 & -1 \\ 2 & 0 & 3 \end{pmatrix}$$

OR: Although this was less popular, for those that chose this option it very often proved a successful end to the paper. Most candidates were able to recall and apply the formula for the shortest distance, and manipulate their expression to reach the given quadratic. They then went on to find the two perpendicular vectors successfully and hence the angle between the two planes, although some calculation errors crept into the evaluation of the cross product.

Answer: $\theta = 66.1°$

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FURTHER MATHEMATICS

Paper 9231/21
Paper 21

Key messages

To score full marks in the paper candidates must be well versed in both Mechanics and Statistics, though any preference between these two areas can be exercised in the choice of the final optional question.

All steps of the argument or derivation should be written down in those questions which require a given result to be shown. Although less detail may suffice in questions where an unknown value or result is to be found, candidates are advised to show sufficient working so that credit may be earned for their method of solution even if an error occurs.

Non-standard symbols introduced by candidates should be defined or explained, for example by showing forces or velocities in a diagram within their written answers. Annotating a diagram in the question paper is insufficient, since this will not be seen by the Examiners.

General comments

Almost all candidates attempted all the questions, and while very good answers were frequently seen, the paper discriminated well between different levels of ability. In the only question which offered a choice, namely **Question 11**, there was a very strong preference for the Statistics option, though some of the candidates who chose the Mechanics option produced good attempts. Indeed all questions were answered well by some candidates, with **Questions 3** and **6** found to be the most challenging.

Advice to candidates in previous reports to set out their work clearly, with any corrections legible and the replacements to deleted attempts readily identifiable was seemingly heeded by many. When relevant in Mechanics questions it is helpful to include diagrams which show, for example, what forces are acting and also their directions as in **Question 11** and the directions of motion of particles, as in **Question 2**. Where the meaning of symbols introduced by candidates is not clearly defined in this way or is not otherwise obvious, it is helpful to include a definition or explanation. This is also true in Statistics questions, and thus in **Questions 7 and 11** any symbols used when stating the hypotheses should either be in standard notation or should be correctly defined.

Many candidates appreciated the need to explain their working clearly in those questions which require certain given results to be verified rather than finding unknown results. It is particularly advisable to explain or justify any new equations which are written down, for example by stating that forces are being resolved in a specified direction or moments are being taken about a specified point. Even when an unknown result must be found, candidates are well advised to show their method, since credit may then be earned for a valid approach if, for example, an incorrect result is due only to an arithmetical error.

The rubric for the paper specifies that non-exact numerical answers be given to 3 significant figures, and while most candidates abided by this requirement, a few rounded intermediate results to this same or lower accuracy with consequent risk that the final result is in error in the third significant figure. Such premature approximations should therefore be avoided, particularly when the difference of two numbers of similar sizes is to be taken, as can happen when taking the difference of means or estimating variances for example.

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Comments on specific questions

Question 1

Few candidates had difficulty with this question. The magnitude of the impulse is found from the change in momentum, while the thickness of the barrier and the time taken to pass through it may be found in several equally valid ways.

Answers: **(i)** 3 N s^{-1}; **(ii)** 0·51 m; **(iii)** 0·003 s.

Question 2

Many candidates were able to formulate and solve correctly two equations for the speeds v_A and v_B of the spheres *A* and *B* after the first collision by means of conservation of momentum and Newton's restitution equation. Collision with the barrier means that the final speed w_B of *B* is $\frac{1}{2}v_B$. Equating $\frac{1}{2}mu^2/5$ to $\frac{1}{2}mv_A^2 + \frac{1}{2}mw_B^2$ leads to a quadratic equation in *e* and hence the required value of *e*.

Answer: 3/5.

Question 3

This question seemingly proved challenging to many candidates, and is one in which it is particularly advisable to think carefully about the given information before embarking on any working. The most obvious approach is to consider the motion of the particle from the point at which its speed equals half its maximum speed to the nearest point at which it reverses direction, expressing this in terms of either the period *T* or the usual parameter ω where $T = 2\pi/\omega$. This is probably most easily achieved by expressing the distance *x* from the centre as 0·25 cos ωt, though 0·25 sin ωt can be used with a little more thought. Candidates must take care over how they relate this time to the given time 4/3 seconds, since the latter refers to a complete oscillation. The appropriate value is therefore ⅓ seconds. An equally valid approach is to consider the time to the mid-point of the motion, which is $\frac{1}{4}T - \frac{1}{3}$ seconds. Having found the value of ω or equivalently *T* in this way, the required maximum speed is the product of ω and the given amplitude 0·25.

Answer: 4 s; $\pi/8$ or 0·393 m s^{-1}.

Question 4

Applying Newton's second law of motion to the particle *P* in a radial direction at *A*, where the reaction becomes zero, gives an equation for cos θ involving *P*'s given speed *v* at *A*, showing it to equal 3/5. A second equation results from conservation of energy from the initial point to *A*, allowing the required speed *u* to be found. The question implies though does not explicitly state that *A* is above the level of *O*, but a minority of candidates wrongly took *A* to be below this level. In the second part many candidates realised that the greatest height subsequently reached above the level of *A* is given by $(v \sin \theta)^2/2g$, to which must be added the height $a \cos \theta$ of *A* above *O*. Attempts based on equating the kinetic energy of *P* at *O* to the gain in potential energy were not successful, since *P* is moving horizontally at its highest point and thus still has some kinetic energy.

Answer: $\sqrt{(19\,ag/5)}$; $99\,a/125$.

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Question 5

The key to finding the required moment of inertia is to not only use standard formulae and the parallel axes theorem to formulate and then sum the individual moments of inertia of the rod and discs, but also to realise that the specified axis is in the plane of the object rather than perpendicular to it. This necessitates use of the perpendicular axes theorem for the discs, so that their individual moments of inertia about the axis are $9{\cdot}25\,ma^2$ and $40\,ma^2$. As in all such questions where a given result must be shown, candidates should include sufficient working so as to justify full credit. Candidates who simply write down a sum of terms with no explanation whatever run considerable risk, since an error in only one term can cast doubt on the validity of their whole process and thereby lose considerable credit. The reference in the final part of the question to the period of small oscillations rightly suggested to most candidates that they should derive an SHM equation. This is readily done by equating the product of the moment of inertia and the angular acceleration to the couple acting on the object when displaced by a small angle θ. Approximating $\sin\theta$ by θ then yields the standard form of the SHM equation and hence the period $T = 2\pi/\omega$.

Answer: $40\pi\sqrt{(a/69g)}$.

Question 6

The first step is to determine the probability p of a score of 6, which can result from 5 rather than 6 of the 36 possible combinations of the two numbers and hence equals 5/36. The required mean then follows from $1/p$. The starting point in the second part is to formulate the inequality $1 - q^{N-1} > 0{\cdot}95$, with $q = 31/36$, solution of which gives $N - 1 > 20{\cdot}03$ and hence the least integral value of N.

Answer: 36/5; 22.

Question 7

Most candidates understood how to carry out this test. As in all such tests, the hypotheses should be stated in terms of the population mean and not the sample mean. The unbiased estimate 63/400 or 0·1575 of the population variance may be used to calculate a t-value of 1·76. Since it is a one-tail test, comparison with the tabulated value of 1·86 leads to acceptance of the null hypothesis, namely that the population mean is not greater than 2·5.

Question 8

As well as integrating $f(x) = 2e^{-2x}$ to find the distribution function $F(x)$, candidates should preferably state that this result holds true for $x \geqslant 0$, and that $F(x) = 0$ for $x < 0$. Apart from the few candidates who confused median and mean, finding the median value m of X by equating $F(m)$ to ½ presented little problem. The first step in the final part is to find or state the distribution function of Y, preferably simplifying it to $1 - 1/y^2$, and this is then differentiated to give the required probability density function.

Answers: **(i)** $0\ (x < 0)$, $1 - e^{-2x}\ (x \geqslant 0)$; **(ii)** ½ ln 2 or 0·347; **(iii)** $0\ (y < 1)$, $2/y^3\ (y \geqslant 1)$.

Question 9

The expected values were usually found correctly from $150\ {}^6C_i\, p^i q^{6-i}$ with $p = 0{\cdot}6$ and $q = 0{\cdot}4$, but not all candidates combined the first two cells to ensure that all the expected values are at least 5. Apart from this, the goodness of fit test was also often carried out well. Comparison of the calculated value 9·13 of χ^2 with the critical value 11·07 leads to acceptance of the null hypothesis, namely that the binomial distribution does fit the data.

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Question 10

Finding the product moment correlation coefficient r is not entirely straightforward in this case, since the values 61·2 and 50·4 of Σx and Σy respectively (or equivalently the sample means) must first be found from the given information, using in particular the standard expression for the variance of sample data. In so doing it is important to distinguish between the exact variance of a sample (as here) which requires division by n = 6 and the estimated population variance which requires instead n –1 = 5. The equations of the two regression lines follow from first finding their gradients and then utilising the sample means found earlier. The value of x when y = 6·4 may be estimated from the regression line of y on x, and a variety of comments are then possible on its reliability.

Answers: **(i)** 0·986; **(ii)** $y = 0{\cdot}507x + 3{\cdot}23$, $x = 1{\cdot}92y – 5{\cdot}91$; **(iii)** 6·36.

Question 11 (Mechanics)

This optional question was attempted by only 15% of the candidates, but many of those who did so made good attempts. The tension T in the string in terms of θ may be found in two ways, namely by taking moments for the rod about A and by applying Hooke's law to the string. Equating the two resulting expressions for T verifies the given value of cos θ and yields the required value of T. The horizontal and vertical components of the reaction force at the hinge may be found by corresponding resolution of forces, and then combined in the usual way to give the required magnitude.
Answer: 3 W; (3/√2) W or 2·12 W.

Question 11 (Statistics)

Most candidates had little difficulty finding the value of Σx from the sample mean, since the latter is the midpoint of the given confidence interval, but Σx^2 proved more challenging. The first step is to recall that the semi-width of the confidence interval is $t\, s_A/\sqrt{12}$, with the tabular value of t here 2·201, from which the unbiased estimate s_A^2 = 1·706 of the population variance of the birds from nature reserve A follows. Recalling that s_A^2 equals $\{\Sigma x^2 – (\Sigma x)^2/12\}/11$ then gives the required value of Σx^2. As in all such tests, the hypotheses in the second part should be stated in terms of the population means and not the sample means. Most of the many candidates attempting this optional question found an unbiased estimate 5·341 of the population variance of the birds from nature reserve B and hence a pooled estimate 2·989 of the common variance. This enables the value 0·869 to be found for t, and comparison with the tabulated value of 1·333 leads to acceptance of the null hypothesis, namely that Petra's belief is unfounded. Since the question states explicitly that the two population variances can be assumed to be equal, it is inappropriate to base the test on an estimate of the variance of the combined populations, as some candidates did.

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FURTHER MATHEMATICS

Paper 9231/22

Paper 22

Key messages

To score full marks in the paper candidates must be well versed in both Mechanics and Statistics, though any preference between these two areas can be exercised in the choice of the final optional question.

All steps of the argument or derivation should be written down in those questions which require a given result to be shown. Although less detail may suffice in questions where an unknown value or result is to be found, candidates are advised to show sufficient working so that credit may be earned for their method of solution even if an error occurs.

Non-standard symbols introduced by candidates should be defined or explained, for example by showing forces or velocities in a diagram within their written answers. Annotating a diagram in the question paper is insufficient, since this will not be seen by the Examiners.

General comments

Almost all candidates attempted all the questions, and while very good answers were frequently seen, the paper discriminated well between different levels of ability. In the only question which offered a choice, namely **Question 11**, there was a very strong preference for the Statistics option, though some of the candidates who chose the Mechanics option produced good attempts. Indeed all questions were answered well by some candidates, with **Questions 3** and **6** found to be the most challenging.

Advice to candidates in previous reports to set out their work clearly, with any corrections legible and the replacements to deleted attempts readily identifiable was seemingly heeded by many. When relevant in Mechanics questions it is helpful to include diagrams which show, for example, what forces are acting and also their directions as in **Question 11** and the directions of motion of particles, as in **Question 2**. Where the meaning of symbols introduced by candidates is not clearly defined in this way or is not otherwise obvious, it is helpful to include a definition or explanation. This is also true in Statistics questions, and thus in **Questions 7 and 11** any symbols used when stating the hypotheses should either be in standard notation or should be correctly defined.

Many candidates appreciated the need to explain their working clearly in those questions which require certain given results to be verified rather than finding unknown results. It is particularly advisable to explain or justify any new equations which are written down, for example by stating that forces are being resolved in a specified direction or moments are being taken about a specified point. Even when an unknown result must be found, candidates are well advised to show their method, since credit may then be earned for a valid approach if, for example, an incorrect result is due only to an arithmetical error.

The rubric for the paper specifies that non-exact numerical answers be given to 3 significant figures, and while most candidates abided by this requirement, a few rounded intermediate results to this same or lower accuracy with consequent risk that the final result is in error in the third significant figure. Such premature approximations should therefore be avoided, particularly when the difference of two numbers of similar sizes is to be taken, as can happen when taking the difference of means or estimating variances for example.

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Comments on specific questions

Question 1

Few candidates had difficulty with this question. The magnitude of the impulse is found from the change in momentum, while the thickness of the barrier and the time taken to pass through it may be found in several equally valid ways.

Answers: **(i)** 3 N s^{-1}; **(ii)** 0·51 m; **(iii)** 0·003 s.

Question 2

Many candidates were able to formulate and solve correctly two equations for the speeds v_A and v_B of the spheres A and B after the first collision by means of conservation of momentum and Newton's restitution equation. Collision with the barrier means that the final speed w_B of B is $\frac{1}{2}v_B$. Equating $\frac{1}{2}mu^2/5$ to $\frac{1}{2}mv_A^2 + \frac{1}{2}mw_B^2$ leads to a quadratic equation in e and hence the required value of e.

Answer: 3/5.

Question 3

This question seemingly proved challenging to many candidates, and is one in which it is particularly advisable to think carefully about the given information before embarking on any working. The most obvious approach is to consider the motion of the particle from the point at which its speed equals half its maximum speed to the nearest point at which it reverses direction, expressing this in terms of either the period T or the usual parameter ω where $T = 2\pi/\omega$. This is probably most easily achieved by expressing the distance x from the centre as $0{\cdot}25 \cos \omega t$, though $0{\cdot}25 \sin \omega t$ can be used with a little more thought. Candidates must take care over how they relate this time to the given time 4/3 seconds, since the latter refers to a complete oscillation. The appropriate value is therefore ⅓ seconds. An equally valid approach is to consider the time to the mid-point of the motion, which is $\frac{1}{4}T - \frac{1}{3}$ seconds. Having found the value of ω or equivalently T in this way, the required maximum speed is the product of ω and the given amplitude 0·25.

Answer: 4 s; $\pi/8$ or 0·393 m s^{-1}.

Question 4

Applying Newton's second law of motion to the particle P in a radial direction at A, where the reaction becomes zero, gives an equation for $\cos\theta$ involving P's given speed v at A, showing it to equal 3/5. A second equation results from conservation of energy from the initial point to A, allowing the required speed u to be found. The question implies though does not explicitly state that A is above the level of O, but a minority of candidates wrongly took A to be below this level. In the second part many candidates realised that the greatest height subsequently reached above the level of A is given by $(v \sin\theta)^2/2g$, to which must be added the height $a \cos\theta$ of A above O. Attempts based on equating the kinetic energy of P at O to the gain in potential energy were not successful, since P is moving horizontally at its highest point and thus still has some kinetic energy.

Answer: $\sqrt{(19\,ag/5)}$; $99\,a/125$.

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Question 5

The key to finding the required moment of inertia is to not only use standard formulae and the parallel axes theorem to formulate and then sum the individual moments of inertia of the rod and discs, but also to realise that the specified axis is in the plane of the object rather than perpendicular to it. This necessitates use of the perpendicular axes theorem for the discs, so that their individual moments of inertia about the axis are $9{\cdot}25\,ma^2$ and $40\,ma^2$. As in all such questions where a given result must be shown, candidates should include sufficient working so as to justify full credit. Candidates who simply write down a sum of terms with no explanation whatever run considerable risk, since an error in only one term can cast doubt on the validity of their whole process and thereby lose considerable credit. The reference in the final part of the question to the period of small oscillations rightly suggested to most candidates that they should derive an SHM equation. This is readily done by equating the product of the moment of inertia and the angular acceleration to the couple acting on the object when displaced by a small angle θ. Approximating $\sin\theta$ by θ then yields the standard form of the SHM equation and hence the period $T = 2\pi/\omega$.

Answer: $40\pi\sqrt{(a/69g)}$.

Question 6

The first step is to determine the probability p of a score of 6, which can result from 5 rather than 6 of the 36 possible combinations of the two numbers and hence equals 5/36. The required mean then follows from $1/p$. The starting point in the second part is to formulate the inequality $1 - q^{N-1} > 0{\cdot}95$, with $q = 31/36$, solution of which gives $N - 1 > 20{\cdot}03$ and hence the least integral value of N.

Answer: 36/5; 22.

Question 7

Most candidates understood how to carry out this test. As in all such tests, the hypotheses should be stated in terms of the population mean and not the sample mean. The unbiased estimate 63/400 or 0·1575 of the population variance may be used to calculate a t-value of 1·76. Since it is a one-tail test, comparison with the tabulated value of 1·86 leads to acceptance of the null hypothesis, namely that the population mean is not greater than 2·5.

Question 8

As well as integrating $f(x) = 2e^{-2x}$ to find the distribution function $F(x)$, candidates should preferably state that this result holds true for $x \geqslant 0$, and that $F(x) = 0$ for $x < 0$. Apart from the few candidates who confused median and mean, finding the median value m of X by equating $F(m)$ to ½ presented little problem. The first step in the final part is to find or state the distribution function of Y, preferably simplifying it to $1 - 1/y^2$, and this is then differentiated to give the required probability density function.

Answers: **(i)** $0\ (x < 0),\ 1 - e^{-2x}\ (x \geqslant 0)$; **(ii)** ½ ln 2 or 0·347; **(iii)** $0\ (y < 1),\ 2/y^3\ (y \geqslant 1)$.

Question 9

The expected values were usually found correctly from $150\ {}^6C_i\, p^i q^{6-i}$ with $p = 0{\cdot}6$ and $q = 0{\cdot}4$, but not all candidates combined the first two cells to ensure that all the expected values are at least 5. Apart from this, the goodness of fit test was also often carried out well. Comparison of the calculated value 9·13 of χ^2 with the critical value 11·07 leads to acceptance of the null hypothesis, namely that the binomial distribution does fit the data.

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Question 10

Finding the product moment correlation coefficient r is not entirely straightforward in this case, since the values 61·2 and 50·4 of $\Sigma\ x$ and $\Sigma\ y$ respectively (or equivalently the sample means) must first be found from the given information, using in particular the standard expression for the variance of sample data. In so doing it is important to distinguish between the exact variance of a sample (as here) which requires division by n = 6 and the estimated population variance which requires instead n –1 = 5. The equations of the two regression lines follow from first finding their gradients and then utilising the sample means found earlier. The value of x when y = 6·4 may be estimated from the regression line of y on x, and a variety of comments are then possible on its reliability.

Answers: **(i)** 0·986; **(ii)** y = 0·507x + 3·23, x = 1·92y – 5·91; **(iii)** 6·36.

Question 11 (Mechanics)

This optional question was attempted by only 15% of the candidates, but many of those who did so made good attempts. The tension T in the string in terms of θ may be found in two ways, namely by taking moments for the rod about A and by applying Hooke's law to the string. Equating the two resulting expressions for T verifies the given value of cos θ and yields the required value of T. The horizontal and vertical components of the reaction force at the hinge may be found by corresponding resolution of forces, and then combined in the usual way to give the required magnitude.
Answer: 3 W; (3/√2) W or 2·12 W.

Question 11 (Statistics)

Most candidates had little difficulty finding the value of $\Sigma\ x$ from the sample mean, since the latter is the midpoint of the given confidence interval, but $\Sigma\ x^2$ proved more challenging. The first step is to recall that thea semi-width of the confidence interval is $t\ s_A/\sqrt{12}$, with the tabular value of t here 2·201, from which the unbiased estimate s_A^2 = 1·706 of the population variance of the birds from nature reserve A follows. Recalling that s_A^2 equals $\{\Sigma\ x^2 – (\Sigma\ x)^2/12\}/11$ then gives the required value of $\Sigma\ x^2$. As in all such tests, the hypotheses in the second part should be stated in terms of the population means and not the sample means. Most of the many candidates attempting this optional question found an unbiased estimate 5·341 of the population variance of the birds from nature reserve B and hence a pooled estimate 2·989 of the common variance. This enables the value 0·869 to be found for t, and comparison with the tabulated value of 1·333 leads to acceptance of the null hypothesis, namely that Petra's belief is unfounded. Since the question states explicitly that the two population variances can be assumed to be equal, it is inappropriate to base the test on an estimate of the variance of the combined populations, as some candidates did.

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FURTHER MATHEMATICS

Paper 9231/23
Paper 23

Key messages

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General comments

Almost all candidates attempted all the questions, and while very good answers were frequently seen, the paper discriminated well between different levels of ability. In the only question which offered a choice, namely **Question 11**, there was a very strong preference for the Statistics option, though some of the candidates who chose the Mechanics option produced good attempts. Indeed all questions were answered well by some candidates, with **Questions 3 and 7** found to be the most challenging.

Advice to candidates in previous reports to set out their work clearly, with any corrections legible and the replacements to deleted attempts readily identifiable was seemingly heeded by many. When relevant in Mechanics questions it is helpful to include diagrams which show, for example, what forces are acting and also their directions as in **Question 2** and the directions of motion of particles, as in **Question 5**. Where the meaning of symbols introduced by candidates is not clearly defined in this way or is not otherwise obvious, it is helpful to include a definition or explanation. This is also true in Statistics questions, and thus in **Questions 8 and 10** any symbols used when stating the hypotheses should either be in standard notation or should be correctly defined.

Many candidates appreciated the need to explain their working clearly in those questions which require certain given results to be verified rather than finding unknown results. It is particularly advisable to explain or justify any new equations which are written down, for example by stating that forces are being resolved in a specified direction or moments are being taken about a specified point. Even when an unknown result must be found, candidates are well advised to show their method, since credit may then be earned for a valid approach if, for example, an incorrect result is due only to an arithmetical error.

The rubric for the paper specifies that non-exact numerical answers be given to 3 significant figures, and while most candidates abided by this requirement, a few rounded intermediate results to this same or lower accuracy with consequent risk that the final result is in error in the third significant figure. Such premature approximations should therefore be avoided, particularly when the difference of two numbers of similar sizes is to be taken, as can happen when taking the difference of means or estimating variances for example.

Comments on specific questions

Question 1

In order to utilise the given relationship between the tensions at the lowest and highest points, each of these tensions must be found in terms of the particle's speed there by use of Newton's law $F = ma$. Candidates should also appreciate that they act in opposite directions at the two points. The two speeds may be related by conservation of energy. The resulting equations may then be solved to find the greatest speed of *P*, namely that at the lowest point.

Answer: $\sqrt{(11ag)}$.

Question 2

Candidates are well advised to first identify all the forces acting on the rod, preferably showing them on a diagram in their answer book so that the symbols used to represent them are clear. This process reveals that three of the forces are unknown, namely the reaction R_A and friction F_A at *A* and the normal reaction R_P at *P*. A further unknown is the required distance *AP*, implying that four independent equations must be formulated in order to find *AP*. One such follows immediately from the information about limiting equilibrium in the question, namely $F_A = \frac{2}{3} R_A$. While there are a variety of possible moment and force resolution equations to choose from, many candidates sensibly resolved forces vertically and horizontally, first replacing F_A and then eliminating R_A to find that $R_P = 5W/3$. Finally taking moments about *A* produces the required distance *AP*. Taking moments about *P* is an obvious alternative, but involves an extra term and hence more possibility of error.

Answer: $9a/5$.

Question 3

Although the value of *c* was usually found correctly by equating the transverse component $2t + c$ of the acceleration when $t = 3$ to the given value 2, finding *d* was markedly more challenging. It is essential to equate the given force magnitude $0{\cdot}4\sqrt{(17)}$ to the resultant of the radial and transverse components of the force acting on *P*, and not just to one component such as the radial one. Working in terms of acceleration rather than force is equally acceptable, but candidates should not confuse linear and angular acceleration; the acceleration of $2\,\mathrm{ms}^{-2}$ mentioned in the question is of course the former. Candidates should also be aware that when taking square roots, as here with $(d - 3)^2 = 4$ or similar, both possible signs should be considered. While it is tempting to dismiss $d - 3 = -2$ since the velocity $t^2 + ct + d$ is then negative when $t = 3$, the information in the question does not preclude this.

Answer: $c = -4$, $d = 1$ and 5.

Question 4

Finding the required moment of inertia *I* presented most candidates with little difficulty, requiring the use of standard formulae and the parallel axes theorem to formulate and then sum the individual moments of inertia of the rod and disc. As in all such questions where a given result must be shown, candidates should include sufficient details of their solution so as to justify full credit. Candidates who simply write down a sum of terms with no explanation whatever run considerable risk, since an error in only one term can cast doubt on the validity of their whole process and thereby lose considerable credit. In the second part of the question the angular speed ω is first found by equating the rotational energy $\frac{1}{2}I\omega^2$ of the object to the change in potential energy as it rotates through an initial angle 60°. The speed which the question requires to be found is by convention the linear speed, however, which is here equal to $6\,a\omega$.

Answer: $\sqrt{(120\,ag/19)}$.

Question 5

Almost all candidates correctly formulated and solved two equations for the speeds of the spheres *A* and *B* after their collision by means of conservation of momentum and Newton's restitution equation, and then repeated the process for the collision between *B* and *C*, hence verifying the given value of *k*. One valid approach to the final part is to find separately the losses in kinetic energy in the two collisions and then sum them, but it is simpler to find the difference between the initial kinetic energy of the particle *A* and the total final kinetic energy. All three particles of course contribute to the latter. The ratio of this loss to the initial kinetic energy of *A* is then found as a percentage.

Answer: $3\,u/2$; **(ii)** 18%.

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Question 6

Most candidates produced good answers to this question, with only the variance of *N* proving problematic for some. It is of course q/p^2 where p is the given probability ¼ and q = ¾. The two required probabilities are found from $1 - q^4$ and q^6 respectively.

Answers: 4, 12; **(i)** 175/256 or 0·684; **(ii)** 729/4096 or 0·178.

Question 7

This seemed to be the most challenging of the compulsory Statistics questions, in part because candidates must relate the given information about the lifetime of life bulbs to the proportions failing and still working after the first 12 months. To estimate the value of λ it is necessary to equate the proportion 0·28 of light bulbs still working after 12 months to 1 – F(12), where F(t) is the cumulative distribution function $1 - e^{-\lambda t}$. The mean μ of T is then $1/\lambda$ and the exponential parameter λ' for the improved light bulbs is equal to $1/1{\cdot}25\mu$. This enables the proportion of the improved light bulbs failing within the first 12 months to be estimated from $1 - e^{-12\lambda'}$, to be stated as a percentage.

Answers: **(i)** 0·106; **(ii)** 9·43; **(iii)** 63·9%.

Question 8

The question does not explicitly specify the use of a particular test, but most candidates realised that a paired-sample *t*-test is appropriate, and conducted the test well. Use of some other test is not acceptable. The first step in the calculation is to find the differences between the pairs of observations and base the test on them. It is course essential to retain the signs of the differences and not just consider their magnitudes. The mean of the resulting sample is then 6/8 and the unbiased estimate of the population variance is 61/50 or 1·22. This gives a calculated value of t of 1·92, and comparison with the tabulated value 2·365 leads to the conclusion of there being no improvement in athletes' times. As in all such tests, candidates should state their hypotheses explicitly in terms of the population rather than the sample means. A further requirement in this case is to make the hypotheses unambiguous; thus $\mu_B > \mu_A$ may not be clear if *A* and *B* are not linked explicitly to *Before* and *After*, for example. Many candidates omitted to state any assumption, the appropriate one here being of the normality of the distributions from which the samples are drawn or indeed the normality of the differences.

Question 9

Most candidates produced good answers to this question, presenting the expected values and their corresponding contributions to the χ^2-value in a clear tabular fashion. After stating the hypotheses to be tested, a contingency table of the expected preferences is produced in the usual way and preferably to an accuracy of two (or more) decimal places. The calculated χ^2-value 6·53 (6·52 is acceptable) should be compared with the tabular value 6·251, leading to acceptance of the alternative hypothesis. Thus there is a gender difference in ice cream flavour preferences.

Question 10

The product moment correlation coefficient r was usually found correctly from the standard formula. Most candidates went on to state the null and alternative hypotheses correctly, which should be in the form $\rho = 0$ and $\rho > 0$, though some wrongly stated them in terms of r which conventionally relates of course to the sample and not the population. Comparison with the tabular value 0·805 leads to a conclusion of there being no evidence of positive correlation. Those candidates who heeded the requirement that the combined sample of 12 be used to find the regression line of y on x usually did so successfully, though some mistakenly used only the original sample of size 5. Finding y from the regression line when $x = 13$ is straightforward, but the result should preferably be given as an integer, both because it is only an estimate and because also the given data suggests that only integer marks are awarded in the tests. A variety of comments on the reliability of the estimate are possible, though one based on a product moment correlation coefficient should find its value for the combined sample rather than relying on the value found earlier for the original sample.

Answers: **(i)** 0·796; **(iii)** p = 167/200 or 0·835, q = 201/22 or 1·00(5); **(iv)** 12.

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Question 11 (Mechanics)

This optional question was attempted by only one-fifth of the candidates, but many of those who did so made good attempts. The length of *AP* when the particle is in equilibrium is found by equating the three forces acting on it, namely the tensions in the two strings and the weight. These forces continue to act on the particle after it is released from rest, and Newton's second law of motion can be applied at a general point to derive an equation in the form $d^2x/dt^2 = -\omega^2 x$ with $\omega^2 = 200/3$, thus verifying simple harmonic motion. The period may then be found from the standard formula $2\pi/\omega$ while the speed v at the mid-point of *AB* follows from $v^2 = \omega^2 (a^2 - x^2)$ with $a = 0{\cdot}4$ and $x = 0{\cdot}35$.

Answers: **(i)** 1·25 m; **(ii)** 0·77 s; **(iii)** 1·58 m s^{-1}.

Question 11 (Statistics)

As in all such tests, the hypotheses should be stated in terms of the population means μ_A and μ_B and not the sample means. The majority of the many candidates attempting this optional question found an unbiased estimate 0·263 of the combined variance s, leading to a calculated z-value of 2·45. Since it is a one-tailed test, comparison with the tabulated value of 2·326 leads to acceptance of the alternative hypothesis, namely that $\mu_B > \mu_A$. Since the question states explicitly that the two population variances cannot be assumed to be equal, it is inappropriate to base the test on a pooled estimate of common variance, as some candidates did. The last part requires the calculation of a similar z-value but here from $1{\cdot}82/2s$ and using the table of the Normal distribution function gives $\Phi(z) = 0{\cdot}962$. Candidates should be aware that a confidence interval implies a two-tailed value of z, so taking α to be 96·2 is incorrect.

Answer: $\alpha = 92{\cdot}4$.

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